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# Measurement of CP violation at the $\Upsilon(4S)$ without time ordering or $\Delta t$

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## Abstract

I derive the expressions for the CP-violating asymmetry arising from interference between mixed and direct decays in the  $\Upsilon(4S)$  system, for the case in which only one of the  $B$  decay times is observed, integrating over the decay time of the other  $B$ . I observe that neither the difference of the decay times  $\Delta t$ , nor even their time-ordering, need be detected. A technique for measurement of the CP-violating weak decay parameter  $\sin 2\beta$  is described which exploits this observation.

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Observed CP violation in the neutral  $K$  system [1] is explained in the Standard Model by the Cabibbo-Kobayashi-Maskawa (CKM) mechanism [2], [3]. Despite 35 years of study, no other system has been found to display CP violation, profoundly limiting the ability to test the CKM model. One of the most important proving grounds for this test is the neutral  $B$  system, which is expected to exhibit large CP asymmetries due to the interference of mixed and direct decays to non-flavor-specific CP eigenstates (henceforth referred to as mixing-induced CP violation). Upcoming asymmetric  $B$  factories, operating at the  $\Upsilon(4S)$ , propose to measure CP asymmetries. These experiments measure flavor asymmetries as a function of the difference of decay times of the two  $B$  mesons from the decay of an  $\Upsilon(4S)$ . This communication examines a new method for use at the  $\Upsilon(4S)$ , in which the decay time of only one  $B$  is observed.

Once the  $\Upsilon(4S)$  has decayed to a pair of neutral  $B$  mesons, the  $B^0\overline{B}^0$  exhibit coherent flavor oscillations until the time of the first meson's decay. Afterwards the second  $B$  meson oscillates freely, with period  $\Delta m$ , from its state at the time of the first  $B$  decay, until its own decay. When one  $B$  decays to a CP eigenstate, CP violation can be established from the other  $B$  meson via a flavor asymmetry in flavor-specific decays. Because the oscillations are coherent, the flavor excess depends on the decay times. Because the meson pair is in a  $C = -1$  state, the excess vanishes if no time information is used. This leads one naturally to consider  $\Upsilon(4S)$  production with asymmetric colliders, where the large boost allows time information to be easily observed.

In the existing literature on  $\Upsilon(4S)$  measurements, observation of the time difference  $\Delta t$  between the two  $B$  decay times is often stated as a necessity. It is not uncommon to find such statements as “to this end [measuring CP violation], one needs to determine the time interval between the two  $B$ -meson decays” [4], “only if we can observe the decay time difference  $T = t - t'$  [ $\Delta t$ ] can we measure a nonzero asymmetry in the decay of a  $C=-1$   $B\overline{B}$  state” [5], or “a determination of  $\Delta t$  is required for the observation of a CP asymmetry in experiments at the  $\Upsilon(4S)$ ” [6]. Several methods have been proposed to measure CP violation using the observed time difference [7,8]. Some work has addressed the case in which the time-ordering is observed but the time difference is not [9], and other work has addressed the case in which the  $b$  quarks are produced incoherently, *e.g.* in  $p\overline{p}$  collisions or at the  $Z^0$  [10,11].

In this communication, I show that for interference between mixed and direct decays to CP eigenstates (such as the “golden mode”  $B \rightarrow \psi K_S$ ), neither time ordering nor time-differences are required in order to be sensitive to CP violation at the  $\Upsilon(4S)$ . This communication focuses on the case for a coherent  $B\overline{B}$  pair when one decay time is measured while the other decay time, and hence even the relative ordering, is unobserved. Specifically, I concentrate on the case in which it is the decay time of the  $B$  decay to  $CP$ -eigenstate that is observed. Typically, this final state contains a  $\psi$  which provides accurate decay time information. Often, the other (flavor-tagging) decay time is difficult to measure accurately; this is the case, for example, when the flavor tag is provided by a charged  $K$  rather than a primary lepton.

I begin with the expression for decay rate of the correlated  $B^0\overline{B}^0$  state into any pair of final states  $f_1$  at time  $t_1$  and  $f_2$  at time  $t_2$ :

$$R(t_1, t_2) = Ce^{-\Gamma(t_1+t_2)}\{(|A_1|^2 + |\overline{A}_1|^2)(|A_2|^2 + |\overline{A}_2|^2) - 4Re(\frac{q}{p}A_1^*\overline{A}_1)Re(\frac{q}{p}A_2^*\overline{A}_2)\}$$

$$\begin{aligned}
& -\cos(\Delta m(t_1 - t_2))[(|A_1|^2 - |\bar{A}_1|^2)(|A_2|^2 - |\bar{A}_2|^2) - 4\text{Im}(\frac{q}{p}A_1^*\bar{A}_1)\text{Im}(\frac{q}{p}A_2^*\bar{A}_2)] \\
& + 2\sin(\Delta m(t_1 - t_2))[\text{Im}(\frac{q}{p}A_1^*\bar{A}_1)(|A_2|^2 - |\bar{A}_2|^2) - (|A_1|^2 - |\bar{A}_1|^2)\text{Im}(\frac{q}{p}A_2^*\bar{A}_2)], \quad (1)
\end{aligned}$$

where I use the notation found in [12], with the standard [13] meanings for  $\lambda$ ,  $p$ , and  $q$ :  $p$  and  $q$  are the coefficients for the quantum-mechanical admixture of  $B^0$  and  $\bar{B}^0$  which comprises the  $B$  mass eigenstates;  $\lambda = \frac{q}{p} \frac{A_{f_{CP}}}{A_{f_{CP}}}$ .  $A_i$  is the amplitude for a  $B^0$  to decay to  $f_i$ , and  $\bar{A}_i$  is the amplitude for  $\bar{B}^0$  to decay to the same state  $f_i$ . As the  $B$  and  $\bar{B}$  have few decay final states in common,  $\frac{\Delta\Gamma}{\Gamma}$  is expected to be small, and has been set to 0 throughout this communication.

To measure CP violation, one observes a decay at time  $t_1$  to a CP eigenstate  $f_{CP}$  and a decay at time  $t_2$  for a flavor-tagging state  $f_{tag}$ . In the case of the flavor-tagging decay of a  $B^0$ ,  $A_2 = A_{tag}, \bar{A}_2 = 0$ :

$$\begin{aligned}
R_+(t_{tag}, t_{CP}) &= Ce^{-\Gamma(t_{tag}+t_{CP})} |\bar{A}_{tag}|^2 |A_{CP}|^2 \\
&\times \{1 + |\lambda_{CP}|^2 - \cos[\Delta m(t_{CP} - t_{tag})](1 - |\lambda_{CP}|^2) \\
&+ 2\sin[\Delta m(t_{CP} - t_{tag})]\text{Im}(\lambda_{CP})\} \quad (2)
\end{aligned}$$

where  $x \equiv \frac{\Delta m}{\Gamma}$ , and I have set  $|\frac{p}{q}| = 1$ , again as expected in the Standard Model. When the flavor tag indicates the decay of the  $\bar{B}^0$  (as opposed to  $B^0$ ) the expression for the rate becomes

$$\begin{aligned}
R_-(t_{tag}, t_{CP}) &= Ce^{-\Gamma(t_{tag}+t_{CP})} |\bar{A}_{tag}|^2 |A_{CP}|^2 \\
&\times \{1 + |\lambda_{CP}|^2 + \cos[\Delta m(t_{CP} - t_{tag})](1 - |\lambda_{CP}|^2) \\
&- 2\sin[\Delta m(t_{CP} - t_{tag})]\text{Im}(\lambda_{CP})\} \quad (3)
\end{aligned}$$

that is, the signs of the cos and sin terms have flipped relative to the case of a  $B^0$  tag. The subscripts on  $R_{\pm}$  serve to denote the flavor tag  $B^0(\bar{B}^0)$ .

By integrating over all times  $t_{tag}$ , while retaining the sign  $(\pm)$  of the flavor tag, one obtains an expression for the decay rate,  $R_{\pm}$ , to a CP eigenstate, as a function of the CP decay time only:

$$\begin{aligned}
R_{\pm}(t_{CP}) &= \int_0^{\infty} dt_{tag} R(t_{tag}, t_{CP}) \\
&= \int_0^{\infty} dt_{tag} Ce^{-\Gamma(t_{tag}+t_{CP})} |\bar{A}_{tag}|^2 |A_{CP}|^2 \\
&\times \{1 + |\lambda_{CP}|^2 \mp \cos[\Delta m(t_{CP} - t_{tag})](1 - |\lambda_{CP}|^2) \\
&\pm 2\sin[\Delta m(t_{CP} - t_{tag})]\text{Im}(\lambda_{CP})\} \\
&= \frac{C|\bar{A}_{tag}|^2 |A_{CP}|^2}{\Gamma(1 + x^2)} e^{-\Gamma t_{CP}} \{(1 + x^2)(1 + |\lambda_{CP}|^2) \\
&\pm (1 - |\lambda_{CP}|^2)[\cos(\Delta m t_{tag}) + x \sin(\Delta m t_{tag})] \\
&\pm 2\text{Im}(\lambda_{CP})[x \cos(\Delta m t_{CP}) - \sin(\Delta m t_{CP})]\} \quad (4)
\end{aligned}$$

Taking  $|\lambda_{CP}|=1$  for the remainder of this communication (i.e., neglecting direct CP violation), the expression for the distribution of decay times to a CP eigenstate is of the form,

$$\begin{aligned}
R_{\pm}(t_{CP}) &= \frac{2C|\bar{A}_{tag}|^2|A_{CP}|^2}{\Gamma(1+x^2)} e^{-\Gamma t_{CP}} \{(1+x^2) \pm \text{Im}(\lambda_{CP})[x \cos(\Delta m t_{CP}) - \sin(\Delta m t_{CP})]\} \\
&= \frac{2C|\bar{A}_{tag}|^2|A_{CP}|^2}{\Gamma} e^{-\Gamma t_{CP}} \{1 \mp \frac{\text{Im}(\lambda_{CP})}{\sqrt{1+x^2}} [\sin(x(\frac{t_{CP}}{\tau} - \frac{\tan^{-1} x}{x}))]\}
\end{aligned} \quad (5)$$

This distribution is shown in figure 1.

Despite having integrated over  $t_{tag}$ , CP asymmetry information is still encoded in the distribution  $R_{\pm}(t_{CP})$ . (Note that the integral  $\int_0^{\infty} dt_{CP} R_{\pm}(t_{CP})$  does equal 0, as expected). Hence even though the decay ordering and time difference are not observed, the decay distributions are different for events that are tagged as  $B^0$  or  $\bar{B}^0$ . This leads to an asymmetry in  $B^0(\bar{B}^0)$  tags as a function of the CP eigenstate decay time,

$$\begin{aligned}
A_{CP}(t_{CP}) &= \frac{R_+(t_{CP}) - R_-(t_{CP})}{R_+(t_{CP}) + R_-(t_{CP})} \\
&= \text{Im}(\lambda_{CP}) \frac{\sin(x(\frac{t_{CP}}{\tau} - \frac{\tan^{-1} x}{x}))}{\sqrt{1+x^2}}
\end{aligned} \quad (6)$$

This distribution is shown in figure 2.

For the CP eigenstate  $\psi K_s$ ,  $\text{Im}(\lambda_{CP})$  is equal to  $\sin 2\beta$ , where  $\beta$  is the usual angle of the unitarity triangle [12], and is nearly equal to  $\arg(V_{td})$ . One measurement method proceeds as follows. The experimenter selects decays in the  $B \rightarrow \psi K_S$  mode. The  $\psi \rightarrow \ell^+ \ell^-$  vertex provides good decay time information when the beam position is well-known. The information is preserved, though diluted, even when only one dimension of the beam is well-determined, as is generally the case in the  $y$  dimension of an  $e^+ e^-$  collider. The other  $B$  decay, with decay time unobserved, provides a tag as either a  $B^0$  or  $\bar{B}^0$ . The decay time distributions for the  $\psi K_S$  candidates are formed for the two tag cases, and fit to the form of equation 5. The lifetimes and mixing parameter  $x$  have been well measured and may be fixed in the fit. The experimenter then fits only for a normalization and the value of  $\sin 2\beta$ . This method is preferable to fitting the asymmetry of equation 6 directly, as resolution effects are more easily incorporated. At a symmetric  $B$  factory, resolutions are expected to be comparable to the lifetime, and therefore important.

Even in the limit of poor resolution, where the oscillation modulations are not directly observable, one may simply measure the mean decay time of the  $B$  CP eigenstate. This mean decay time will be different for events in which the other  $B$  is tagged as a  $\bar{B}^0$  or  $B^0$  decay:

$$\bar{t}_{\pm} = \tau \left( 1 \mp \frac{x \sin 2\beta}{(1+x^2)^2} \right) \quad (7)$$

The difference in the mean times is proportional to  $\sin 2\beta$ :

$$\bar{t}_+ - \bar{t}_- = \tau \frac{2x \sin 2\beta}{(1+x^2)^2} \quad (8)$$

For  $x = 0.7$ ,  $\Delta \bar{t} = 0.63 \tau \sin 2\beta$ . In the  $\Upsilon$  rest frame this is a difference of  $18 \times \sin 2\beta \mu\text{m}$ .

The technique of integrating over one decay time may also be applied to the CP side. That is, CP violation may be established by observing the flavor-tagging  $B$ 's decay time

while integrating over the CP tag decay time. This makes it possible to observe indirect CP violation in all-neutral final states. Previously it was often assumed that these modes could be useful only for measuring decay rates and direct CP violation. Examples of such modes include  $\pi^0\pi^0$ ,  $K_S\pi^0$ , or  $K_SK_S$ . These modes are expected to have very small branching fractions, but may be helpful in understanding penguin contributions to the measurement of  $\sin 2\alpha$ .

Many of the remarks and conclusions concerning CP asymmetries apply also to a measurement of  $\Delta m$ . To measure  $\Delta m$ , the two decays observed are both flavor-tagged. As with CP violating parameters,  $\Delta m$  may be measured using events in which one decay time is observed and the other unobserved. Equation 1 is again the starting point. For events observed to be unmixed (two flavor tags of opposite flavor),  $A_1 = A$ ,  $\overline{A}_1 = 0$ ,  $\overline{A}_2 = 0$ , and  $A_2 = A^*$ . Performing the integration over one of the times gives

$$R_{unmixed}(t) = 2C|A|^4 \frac{e^{-\Gamma t}}{\Gamma(1+x^2)} \{1 + x^2 + [\cos(\Delta mt) + x \sin(\Delta mt)]\} \quad (9)$$

For the rate to mixed events (both flavor tags of the same sign),  $A_1 = A$ ,  $\overline{A}_1 = 0$ ,  $\overline{A}_2 = A^*$ , and  $A_2 = 0$ :

$$R_{mixed}(t) = 2C|A|^4 \frac{e^{-\Gamma t}}{\Gamma(1+x^2)} \{1 + x^2 - [\cos(\Delta mt) + x \sin(\Delta mt)]\} \quad (10)$$

The distributions  $R_{\pm}$  are shown in figure 3.

The resulting asymmetry of mixed to unmixed events is thus

$$\begin{aligned} A_{unmixed/mixed}(t) &= \frac{\cos(\Delta mt) + x \sin(\Delta mt)}{1+x^2} \\ &= \frac{\cos(x(\frac{t}{\tau} - \frac{\tan^{-1} x}{x}))}{\sqrt{1+x^2}} \end{aligned} \quad (11)$$

This asymmetry, along with the CP asymmetry, is shown in figure 2.

Again, the average decay times are different for the two cases of mixed and unmixed events,

$$\bar{t}_{u/m} = \tau(1 \pm \frac{x^2}{(1+x^2)(1 \mp 1+x^2)}) \quad (12)$$

and mean decay time difference

$$\Delta\bar{t} = \tau \frac{2}{1+x^2} \quad (13)$$

The form of the asymmetry in equation 11 makes clear the form of the asymmetry in equation 6, and also provides an interpretation of the physics. The asymmetry 11 is offset from the familiar  $\cos(\Delta mt)$  form by a time phase of  $\frac{\tan^{-1} x}{x}$  lifetimes, which is nearly 1 for small values of  $x$ <sup>1</sup>. That is, the mixing oscillation acts nearly as if the other  $B$  had decayed

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<sup>1</sup>The time phase is 0.87 lifetimes for  $x=0.7$ .

at time  $t \simeq \tau$ . A slight ( $\simeq 20\%$ ) dilution arises from the  $\frac{1}{\sqrt{1+x^2}}$  term. Because it arises from interference between mixing and direct decays, CP violation carries the same phase.

In order to estimate the experimental sensitivity to the CP violation parameter  $\sin 2\beta$  at a symmetric  $B$  factory, one must make assumptions about decay resolutions, reconstruction efficiencies, and effective flavor tagging efficiency. Expected decay lengths for  $B^0(\overline{B^0})$  tagged  $B \rightarrow \psi K_S$  decays are  $34(22)\mu\text{m}$  at a symmetric  $\Upsilon(4S)$  machine, for  $\sin 2\beta = 0.7$  and  $c\tau_B = 468\mu\text{m}$ . The average  $y$  projections of these decay lengths are  $19(12)\mu\text{m}$ . With an integrated luminosity of  $30 \text{ fb}^{-1}$ , effective effective flavor-tagging efficiency of 0.35, and average  $y$ -vertex resolution of  $25\mu\text{m}$ , Monte Carlo studies indicate that the  $\psi K_S$  mode provides a measurement of  $\sin 2\beta$  with statistical uncertainty  $\pm 0.37$ . The CLEO II.V detector, in its measurements of the  $D$  lifetimes [14], has demonstrated the ability to control systematic uncertainties to a few  $\mu\text{m}$ . Due to the small decay lengths, sensitivity is nearly linear in the vertex resolution, in contrast to an asymmetric  $B$  factory.

In conclusion, I have shown that CP violation and mixing are both observable at the  $\Upsilon(4S)$  without the need for time ordering of the  $B$  decays or measurement of  $\Delta t$ . I have presented estimates of the required luminosity for precision measurements of  $\sin 2\beta$ .

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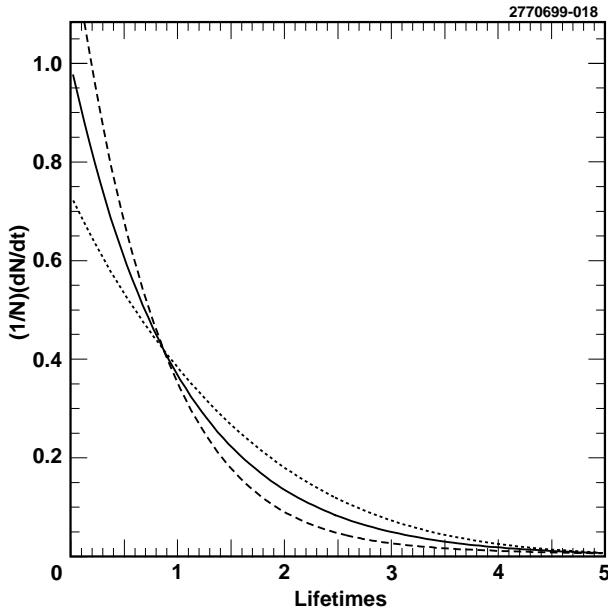


FIG. 1. Effect of CP violation on decay time distributions of  $\Psi K_S$  final state. The solid line shows the case  $\sin 2\beta=0$ ; the dotted(dashed) line shows the distribution of the decay times when the opposite  $B$  is tagged as a  $B^0(\overline{B}^0)$ , for  $\sin 2\beta=0.7$ .

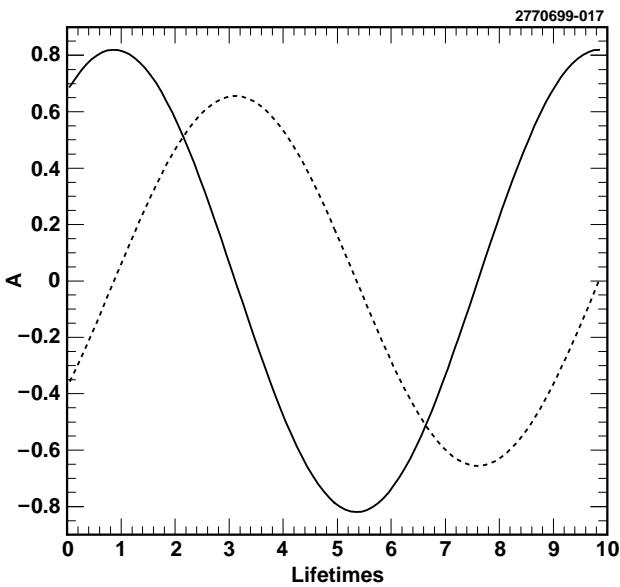


FIG. 2. The asymmetries for  $A_{CP}$  (dashed) and  $A_{mixed/unmixed}$  (solid) and CP tags (dashed) of the measured decay time. The plots are generated setting  $x=0.7$ ,  $\sin 2\beta=0.7$ .

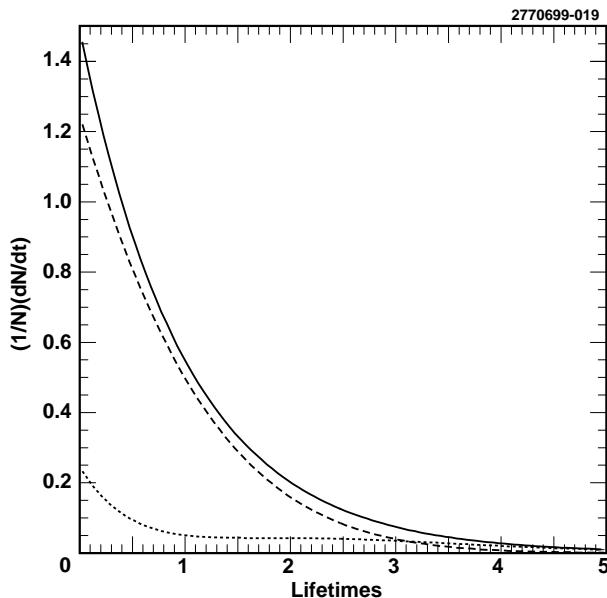


FIG. 3. Effect of  $B - \bar{B}$  mixing on decay time distributions on a flavor-tagging final state. The solid line shows the exponential decay in the case of no mixing ( $x_d = 0$ ). The dotted(dashed) line shows the distribution of the decay times when the opposite  $B$  is tagged as a mixed(unmixed), for  $x_d=0.7$ . The large average decay time of mixed events is readily apparent from the plot.